4.3 Residual Strength Capability

To establish the residual strength capability of a given structure under certain loading conditions, prediction techniques must be developed with a thorough understanding of the complexities involved in evaluating the residual strength. For monolithic or single load path structures which must be classified as slow crack growth structures, the estimation of residual strength capability is straightforward. In multiple load path, built-up structures, whether classified as slow crack growth or fail-safe structures, the strength analysis can become complicated due to the complex geometric construction of the built-up components. In general, the prediction techniques are based on the critical value of the stress-intensity factor for a given geometry and loading. Using fracture toughness failure criteria as explained earlier, the decay in critical stress can be obtained in terms of crack size.

As described by Figure 4.1.2, the residual strength capability is a function of service time for a given structure. This is because the residual strength capability depends on the size of the crack in the structure and the crack grows as a function of time. Thus, to obtain a residual strength capability curve (Figure 4.1.2), one needs two types of data: (a) the relationship between crack length and time, and (b) the relationship between fracture strength (σ_f) and crack length. Section 5 is devoted to obtaining the crack length-time relationship and the remainder of this section is devoted to presenting methods and procedures for obtaining the fracture strength-crack length (σ_f vs. a) relationship. It is to be noted that the σ_f vs. a relationship is independent of time and has been referred to in the general literature as the residual strength diagram. This section presents useful information about residual strength diagrams for single load path and for multiple load path structures.

4.3.1 Single Load Path Residual Strength Diagrams

For a single load path structure, such as an unstiffened panel, the residual strength diagram under plane strain conditions, consists of a single curve as shown in Figure 4.3.1. The procedure for developing the residual strength diagram involves the calculation of the critical stress σ_f, for the critical crack length a_c, using the relationship

$$K_{cr} = \sigma_f \beta \sqrt{\pi a_c}$$

where K_{cr} is the known value of fracture toughness of the material. (K_{cr} may be equal to K_{Ic} or K_c depending on the problem.) The plot of σ_f vs. a_c then provides the necessary residual strength diagram required in design analysis for the simple configuration.

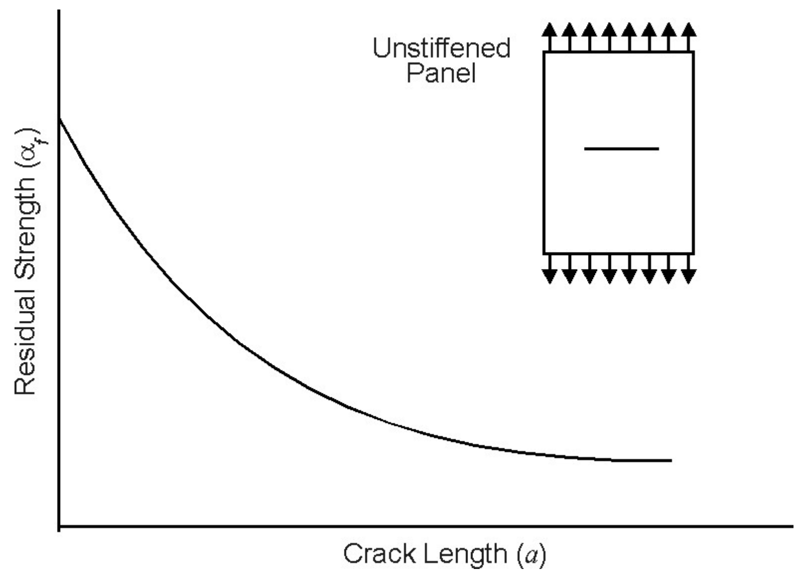


Figure 4.3.1. Residual Strength Diagram for Abrupt Failure of a Single Load Path Structure

The available fracture mechanics solution techniques, as given in Section 11, can be employed in the calculation of the crack-tip stress-intensity factor K to construct the residual strength diagram. Depending on the complexity of the structure, K can be calculated either numerically or through closed form solutions. These techniques, in conjunction with an appropriate failure criterion, can then be used to determine the residual strength capabilities of a given structure.

In general, the construction of a residual strength diagram involves three steps:

(a) The development of the relationship between the applied stress σ, the crack length parameter a, and the applied stress-intensity factor K for the given structural configuration (see Section 11).

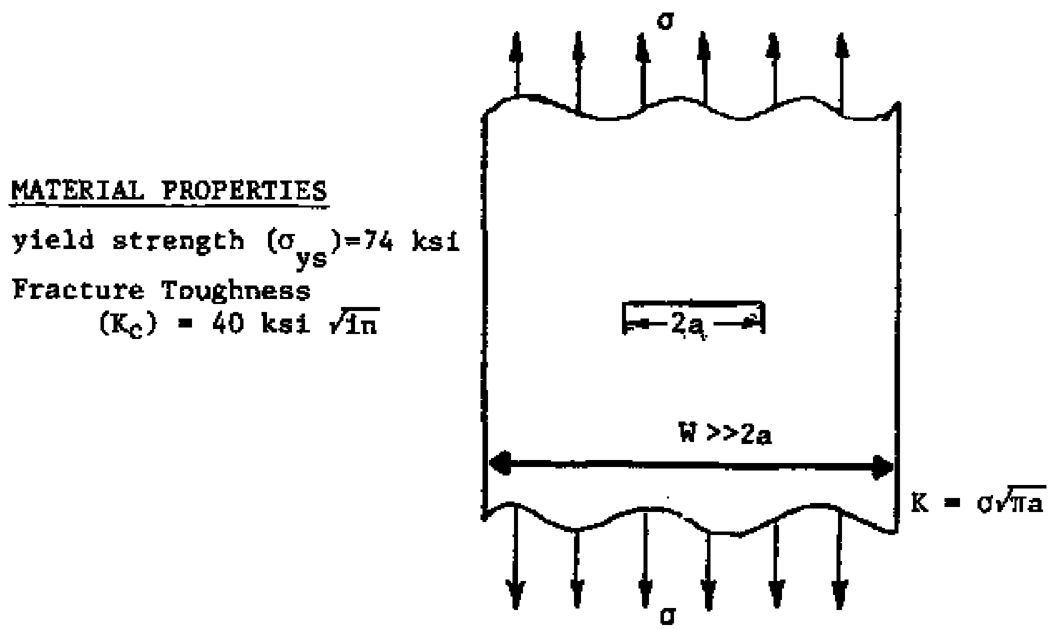
(b) The selection of an appropriate failure criterion based for the expected material behavior at the crack tip (see Section 4.2.1).

(c) The fracture strength (σ_f) values for critical crack sizes (a_c) are obtained utilizing the results of the first two steps and residual strength diagram (σ_f vs. a_c) for the given structural configuration is plotted.

To understand these three steps for constructing a residual strength diagram, the following example is considered. The example considers a wide thin panel with a central crack that has a simple relationship for the stress intensity factor. This example illustrates the importance of the stress-intensity factor for constructing the residual strength diagram.

EXAMPLE 4.3.1 Unstiffened Center Crack Panel

Construct the residual strength diagram for the wide unstiffened panel shown here, assuming that the structure is made from 7075-T6 aluminum sheet material, with a fracture toughness of 40 ksi $\sqrt{in.}$



SOLUTION:

Step 1. Define the stress-intensity factor relationship. From Section 11, the stress intensity factor for a wide unstiffened, center crack panel is given by

$$K = \sigma\sqrt{\pi a}$$

Step 2. Define the failure criterion. For this problem, it is assumed that an abrupt fracture occurs and the condition that defines the fracture is

$$K = K_{cr} = K_c = 40\ ksi\sqrt{in}$$

Step 3. Utilize the results of the first two steps to derive a relationship between fracture strength (σ_f) and critical crack size (a_c), the σ_f vs. a relationship is given by

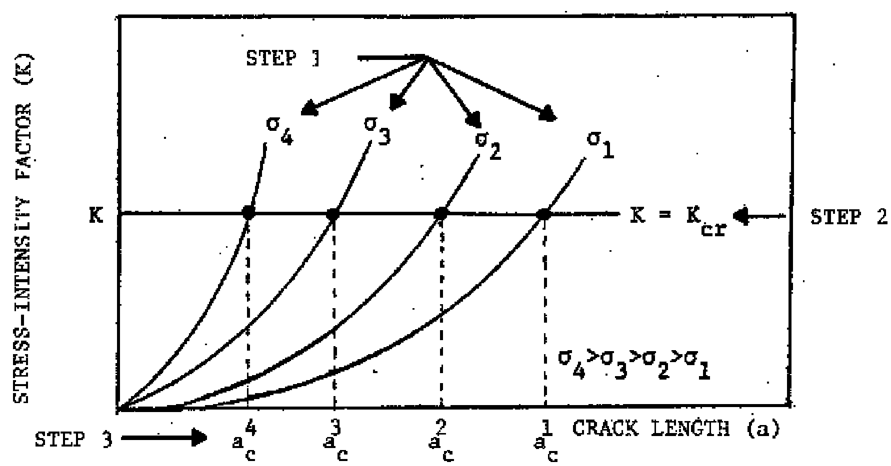
$$\sigma_f\sqrt{a_c} = 40/\sqrt{\pi}$$

For a half crack size (a_c) of 2.0 inch, the fracture strength (σ_f) is about 16 ksi. Other (σ_f vs. a_c) values can be similarly obtained. Once a sufficient number of values are available, the residual strength diagram can be developed, or one could also attack the problem in the graphic manner that is explained using the following:

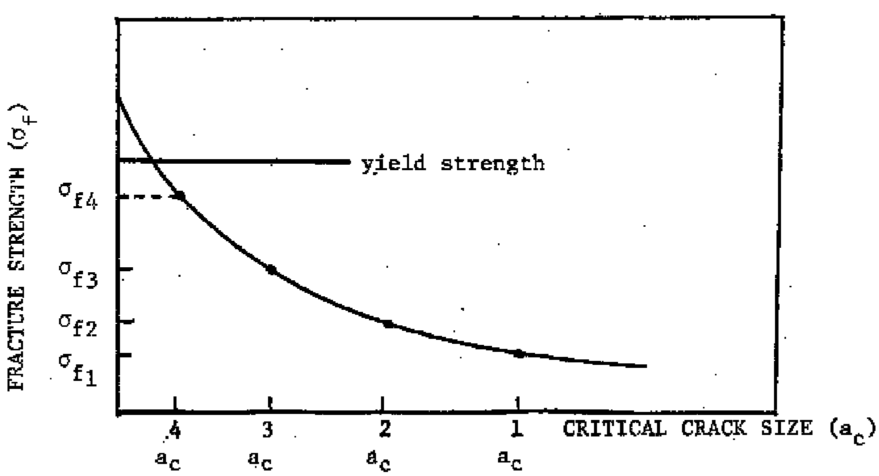
Step 1. Construct a plot of K vs a by using the equations in Step 1 for various values of stress and crack lengths.

Step 2. Superimpose the horizontal line $K = K_{cr} = 40$ ksi $\sqrt{in}$ on the diagram. This line represents the critical stress intensity, i.e., fracture toughness, for this material and is independent of crack length.

Step 3. Complete the residual strength diagram. Utilize the intersection points of the horizontal line with curves where the failure criterion is satisfied, i.e. where $K_{cr} = \sigma_f \sqrt{\pi a_c}$. The values of the respective stresses and the crack sizes at these points are termed to be the failure stresses and the critical crack sizes for the given structure, i.e., the unstiffened panel. The residual strength diagram is finally constructed by plotting the σ_f vs. a_c curve.



(a) Stress-Intensity Factor as a Function of Crack Length for Constant Values of Stress



(b) Residual Strength Diagram

4.3.2 Built-Up Structure Residual Strength Diagrams

In single load path structures, the residual strength analysis involved only one failure criterion for a given structural geometry. In built-up structures, due to the complex geometrical configuration, one or more failure criterion may have to be considered in the determination of residual strength for the whole structure. The following paragraphs examine these aspects of the residual strength analysis of built-up structures.

It was explained earlier that safety can be achieved by designing aircraft structure either as slow crack growth or as fail-safe. The latter case can further be classified into two cases: Multiple Load Path and Crack Arrest. Typically, both Multiple Load Path and Crack Arrest structures are built-up structures. In Section 1.3, the definitions and requirements for these two types of built-up structure are discussed. For completeness, the structure shown in Figure 4.3.2 is analyzed to further explain the features inherent in multiple load path, built-up structure.

As long as the central member is not failed, all three elements carry a share of the total load *P*. In the event of failure of the center member, the total load *P* (actually 1.15*P*) must be transmitted by the other two members at the instant of failure, if the structure is to stay intact.

The residual strength capability for the multiple load path structure shown in Figure 4.3.2 can be explained with Figure 4.3.3. When one element fails, Figure 4.3.3 shows that the remaining parallel members are able to carry the required load without failure. The residual capability is shown to degrade as the crack in the central member extends and as the cracks in the remaining elements fail. Figure 4.3.3 shows the discontinuous change in the strength capability as a result of element failures. Since the load levels in other members dramatically increase, if the load *P* must be maintained, the remaining members will have short lives. Thus, the second member may fail after the time (t_2). The residual strength capability is shown to drop below the safe level somewhere in time between t_1 and t_2. The duration of the time interval between the failure of the first element and the failure of the structure may be short or long depending on the "type of failure" of the first member and the load requirements subsequent to this failure. This time interval is available for the detection of the failure of the first member and the repair of the structure.

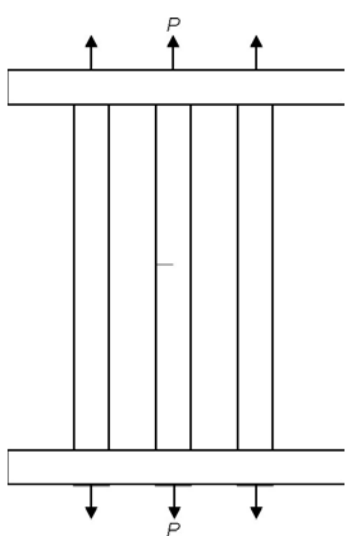


Figure 4.3.2. Multiple Load Path (Built-up) Structure with a Crack in the Central Member

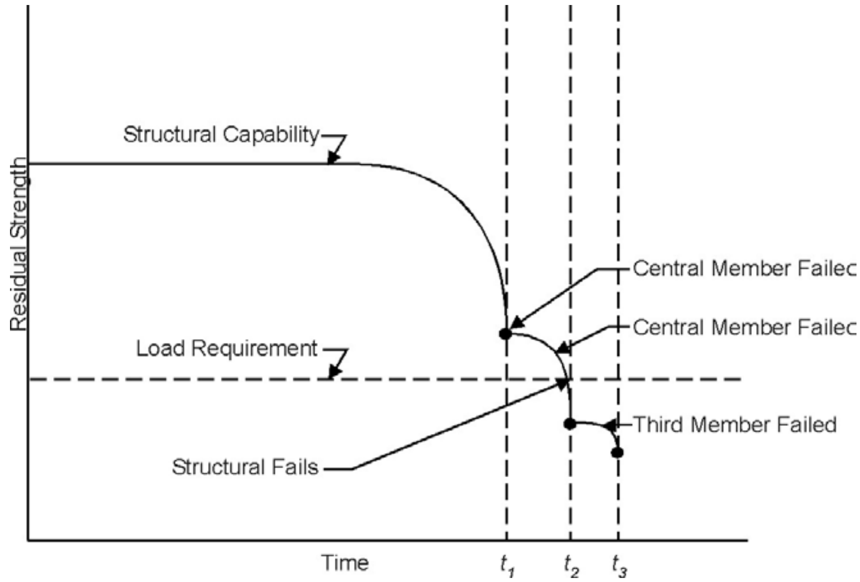


Figure 4.3.3. Reduction of Residual Strength During Successive Failure of Members in the Structure Shown on Figure 4.3.2

The failure stress or the critical flaw size level of the central member (any one of the parallel members) can be estimated by treating the problem in a manner similar to the single load path structure. Using a fatigue crack growth analysis, the crack propagation curve is obtained from the minimum detectable crack size to the critical crack length as illustrated in Figure 4.3.4. In multiple load path structure, partial failure of the structure can occur during its operating period. But this failure must be detected at an inspection before catastrophic failure of the entire structure occurs. A suitable inspection schedule must include analysis of structural characteristics along with the operational requirements for the intervals between inspections.

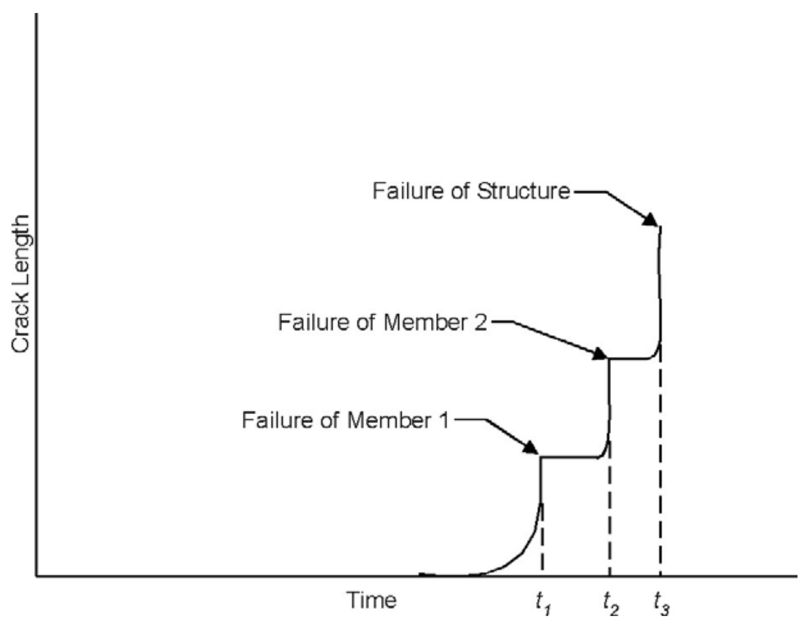


Figure 4.3.4. Crack Growth for Multiple Load Path Structure Shown in Figure 4.3.2

To illustrate the analysis involved in the estimation of residual strength of complex structures, consider an axially loaded skin-stringer combination with longitudinal stiffening as shown in Figure 4.3.5. Assuming that the fasteners are rigid, the displacements of adjacent points in skin and stringers will be equal. (If skin and stringers are made from the same material, the stresses in the two will also be equal for the case of no crack.) Let a transverse crack develop in the skin. This will cause larger displacement in the skin, and the stringers must follow this larger displacement. As a result, they take load from the skin, thus decreasing the skin stress at the expense of higher stringer stress. Consequently, the displacements in the cracked skin will be smaller than in an unstiffened plate with the same size of crack. This implies that the skin stresses are lower and that the stress-intensity factor is lower. The closer the stringers are to the crack, the more effective is the load transfer.

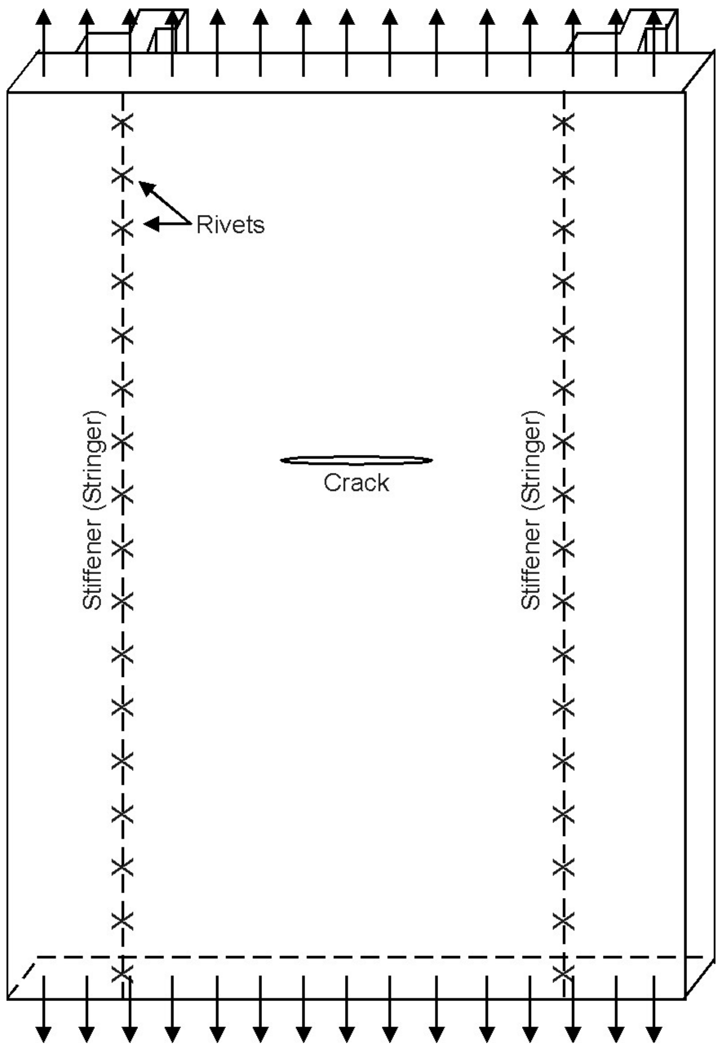


Figure 4.3.5. Skin-Structure Built-Up Structure

If the stress-intensity factor for a small crack in an unstiffened panel is approximated by $K = \sigma\sqrt{\pi a}$, the stress-intensity factor for the stiffened plate will be $K = \beta\sigma\sqrt{\pi a}$. The reduction factor, $\beta = K / \sigma\sqrt{\pi a}$, will decrease when the crack tip approaches a stringer. Since the stringers take load from the skin, the stringer stress will increase from σ to $L\sigma$, where L increases as the crack tip approaches the stringer. Obviously, $0 < \beta \leq 1$, and $L \geq 1$. These values depend upon stiffening ratios, the stiffness of the attachment, and the ratio of crack size to stringer spacing. As will be shown subsequently, β and L can be readily calculated; at this point it is sufficient to note that β and L vary with crack length as shown in Figure 4.3.6.

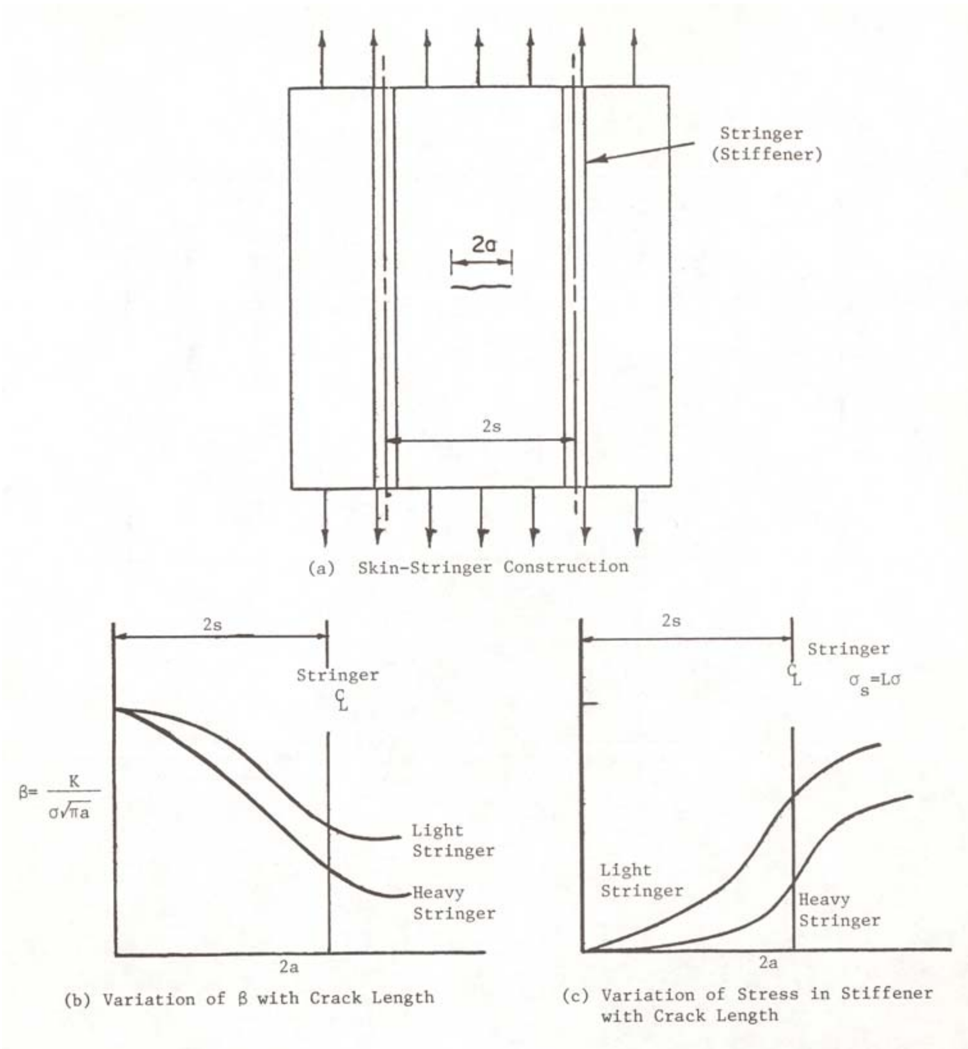


Figure 4.3.6. Variation of *B* and *L* with Crack Length in Stiffened Panel with a Crack Between the Stiffeners

Due to the complexity of stiffened skin structure, the construction of a residual strength diagram is considerably more difficult. Consider first the condition where an abrupt failure in the skin occurs. When the crack is small as compared to the stiffener spacing, the residual strength of the skin is not influenced by the stiffeners and the initial portion of the diagram follows the plot for an unstiffened panel (see point A in Figure 4.3.7). Once the crack size is long enough that the skin cannot sustain the applied load any further, the stringer will take some of the load from the skin, thus decreasing the skin stress. Consequently, the crack-tip stress-intensity factor will be lower due to the reduced stress and so the residual strength of the skin structure will increase with crack length as shown in Figure 4.3.6. As the crack size increases further toward the stiffener location, the load transferred from the skin to the stiffener also increases significantly, thus reducing the stress-intensity factor. The residual strength of the stiffened panel continues to increase as shown in the figure for longer cracks. It can also be noted from the figure that the residual strength diagram for an unstiffened panel would have followed the dotted line, i.e., the continuous decay in the residual strength as the crack size increases. This is because there is no inherent feature present in the single load path structure to decrease the crack tip stress-intensity factor.

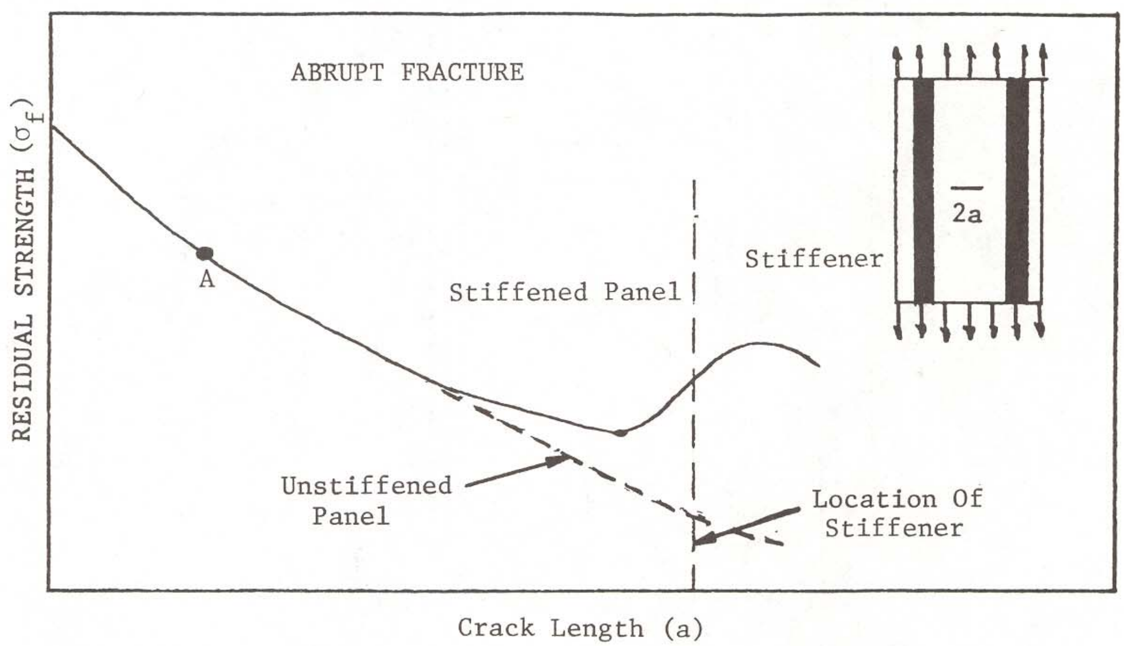


Figure 4.3.7. Residual Strength of the Cracked Panel as a Function of Crack Length for Built-Up Skin-Stiffened Structure Compared with Unstiffened Panel. Abrupt Failure Criterion Used to Determine Residual Strength

The residual strength diagram for the skin-stiffened structure is repeated in Figure 4.3.8 where several additional points of interest are defined for the analyst. For a structure with a crack of length a_A, the residual strength is identified as point A. Since point A is associated with a failure stress that is above the peak stress (σ_{peak}), the crack extends abruptly and completely fails the panel. If the structure contains a crack of length a_C, in the range between a_B and a_D, the crack extends abruptly but then arrests at crack length a_E, where the residual strength available is greater than the applied (failure) stress. This crack extension and arrest feature of skin-stringer construction greatly facilitates meeting inspection requirements for fail-safe structures.

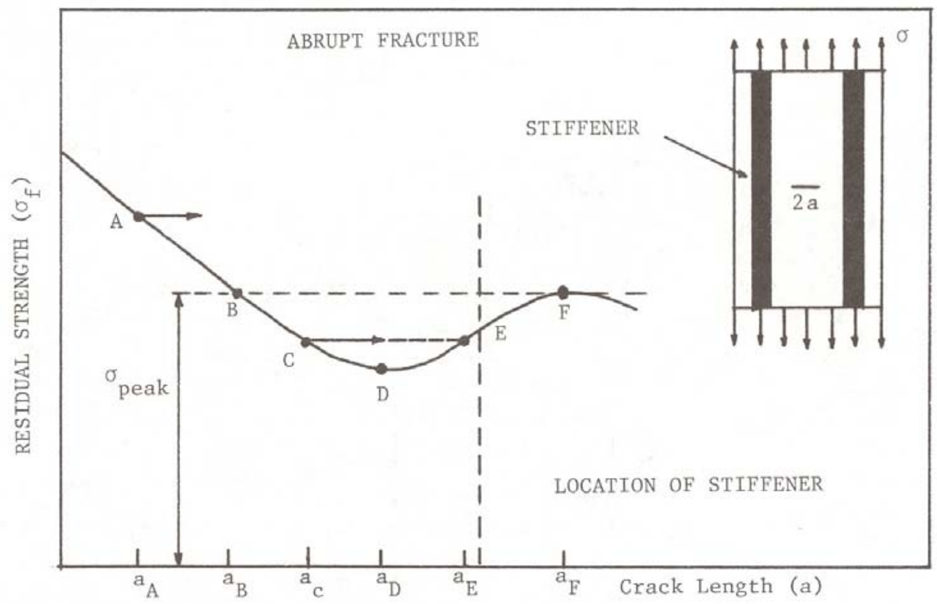


Figure 4.3.8. Residual Strength of the Cracked Panel as a Function of Crack Length for Built-Up Skin Stiffened Structure. Only Skin Failure Mode Considered. Abrupt Failure Criterion Used to Determine Residual Strength

Before the panel fails completely, the failure stress level at point C/E must be increased to the level associated with point F, i.e. to σ_{peak}. As the stress is increased above the level of point E, the crack extends from a_E to maintain an equilibrium between the input stress and the residual strength. When the stress reaches σ_{peak}, the crack has extended to a_F, at which point the crack abruptly extends causing failure of the panel. A schematic illustrating the load crack length diagram observed during an abrupt crack extension/arrest situation in a skin-stringer structure is presented in Figure 4.3.9. Thus, it is seen that the residual strength curve ABCDEF shown in Figure 4.3.8 can be replaced for all practical purposes with a curve that connects points ABF.

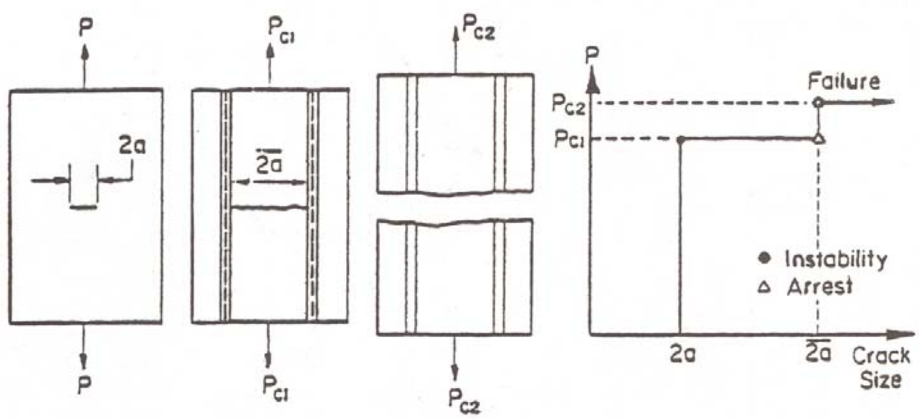


Figure 4.3.9. Load-Crack Length Behavior Observed in Skin-Stiffened Construction with Arrest Features

In the design of fail-safe structure, a frequent objective is to design the structure for limiting or arresting unstable crack growth so that catastrophic failure can be prevented. A number of arrest techniques are described in Bluhm [1969], Romauldi & Sanders [1959-1960] and Broek [1974]. The fundamental concept in crack arrest design is to provide within the structure a means to reduce the crack tip stress intensity factor. This concept requires the use of additional stiffening members such as stiffeners, reinforcing rings, etc., to produce a decrease in the stress. These are inherently present in built-up structures, such as aircraft wings, fuselages, etc..

In general, the residual strength analysis of a structure with crack arrest capabilities may involve more than one failure criterion. For instance, in a stiffened skin structure or an aircraft wing, the analysis should consider stringer failure, fastener failure, and skin crack failure criteria. Built-up panels loaded to fail-safe levels tend to exhibit substantial local deformations of critical elements. Failure criteria are thus dependent also on elastic-plastic deflection allowables for both fastener and skin/stringer elements. Gunther and Wozumi [1982] provide additional details on the residual strength analysis of complex panels based on the ultimate stringer strain.

The residual strength diagram for the structure that exhibits slow crack growth behavior will contain two curves as shown in Figure 4.3.10. The lower curve corresponds to the critical level of stress at which slow crack extension starts. The onset of slow tearing is then described by this lower curve. The upper curve provides the critical stress level at which the unstable rapid crack extension occurs. When the crack approaches the stiffener, as explained earlier, the residual strength levels, corresponding to the onset of slow cracking and the rapid extension, start increasing.

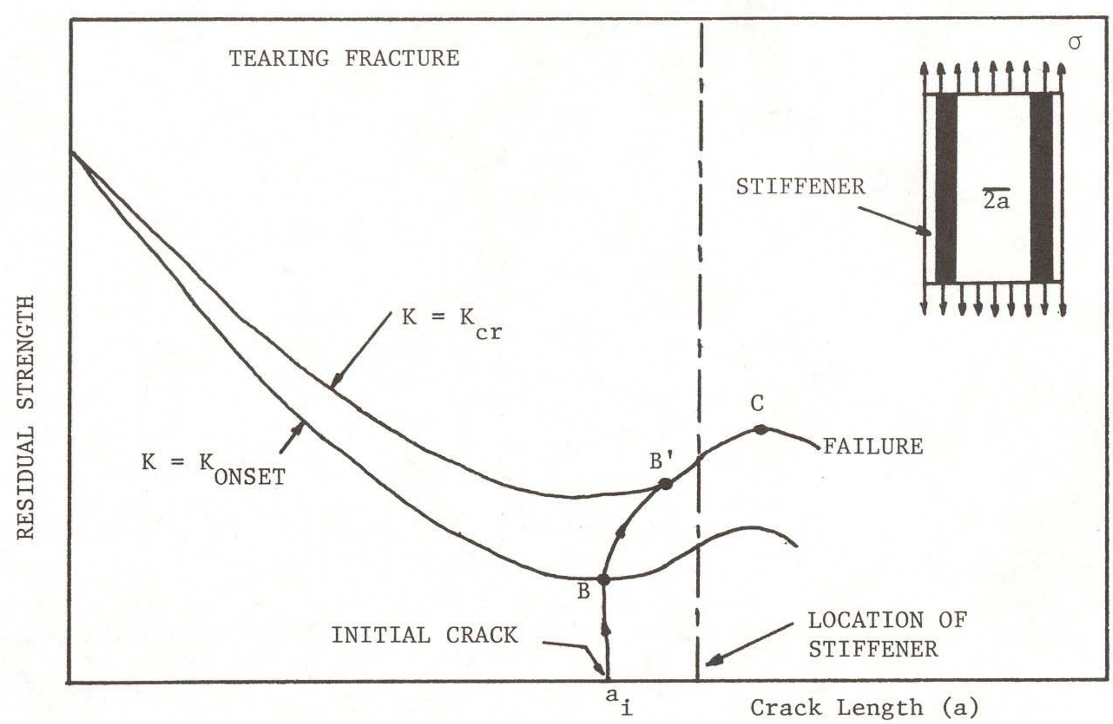


Figure 4.3.10. Residual Strength of Cracked Panel as a Function of Crack Length for Built-up Skin-Stringer Structure. Tearing Failure Criterion Used to Determine Residual Stress

For a crack length a_i, as shown in Figure 4.3.10, the slow crack extension begins at point B. This stable extension continues up to point B′ where the rapid failure is supposed to occur. However, due to the continuous rise in the residual strength of the stiffened panel, the stable crack extension continues to occur beyond point B′ and up to point C. Since the residual strength of the panel starts reducing at this point, any further increase in the applied load will lead to the rapid unstable crack extension.

The construction of the residual strength diagram follows the three steps presented in Section 4.3.1. Due to the complexity of the structural geometry, however, estimating requires the calculation of the loads that are transferred to the stiffening or secondary members from the main load carrying member of the structure. Depending upon the complexity, the K vs. a curves can be obtained either through an appropriate numerical method or through the method of superposition. The methods for constructing residual strength diagrams and for the residual strength capability analyses are further discussed in the following sections with various example problems.